Exercise 49

Find the horizontal and vertical asymptotes of each curve. If you have a graphing device, check your work by graphing the curve and estimating the asymptotes.

$$y = \frac{2x^2 + x - 1}{x^2 + x - 2}$$

Solution

Calculate the limits as $x \to \pm\infty$ to determine the horizontal asymptote. In the second limit, make the substitution, $x = -u$, so that as $x \to -\infty$, $u \to \infty$.

$$\lim_{x\to\infty} \frac{2x^2 + x - 1}{x^2 + x - 2} = \lim_{x\to\infty} \frac{2 + \frac{1}{x} - \frac{1}{x^2}}{1 + \frac{1}{x} - \frac{2}{x^2}} = \frac{2 + 0 - 0}{1 + 0 - 0} = 2$$

$$\begin{aligned}
\lim_{x\to-\infty} \frac{2x^2 + x - 1}{x^2 + x - 2} &= \lim_{u\to\infty} \frac{2(-u)^2 + (-u) - 1}{(-u)^2 + (-u) - 2} \\
&= \lim_{u\to\infty} \frac{2u^2 - u - 1}{u^2 - u - 2} \\
&= \lim_{u\to\infty} \frac{2 - \frac{1}{u} - \frac{1}{u^2}}{1 - \frac{1}{u} - \frac{2}{u^2}} \\
&= \frac{2 - 0 - 0}{1 - 0 - 0} \\
&= 2
\end{aligned}$$

Therefore, the horizontal asymptote is $y = 2$. The vertical asymptotes are found by setting what's in the denominator equal to zero and solving for x.

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

$$x = -2 \quad \text{or} \quad x = 1$$

The function is graphed versus x below with the asymptotes labelled.