Exercise 49

Find the horizontal and vertical asymptotes of each curve. If you have a graphing device, check your work by graphing the curve and estimating the asymptotes.

$$y = \frac{2x^2 + x - 1}{x^2 + x - 2}$$

Solution

Calculate the limits as $x \to \pm \infty$ to determine the horizontal asymptote. In the second limit, make the substitution, x = -u, so that as $x \to -\infty$, $u \to \infty$.

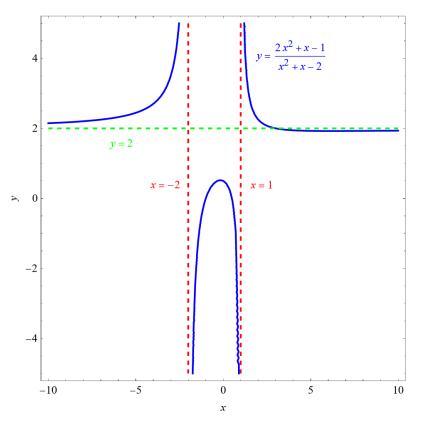
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$$\lim_{x \to \infty} \frac{2x^2 + x - 1}{x^2 + x - 2} = \lim_{x \to \infty} \frac{2 + \frac{1}{x} - \frac{1}{x^2}}{1 + \frac{1}{x} - \frac{2}{x^2}} = \frac{2 + 0 - 0}{1 + 0 - 0} = 2$$
$$\lim_{x \to -\infty} \frac{2x^2 + x - 1}{x^2 + x - 2} = \lim_{u \to \infty} \frac{2(-u)^2 + (-u) - 1}{(-u)^2 + (-u) - 2}$$
$$= \lim_{u \to \infty} \frac{2u^2 - u - 1}{u^2 - u - 2}$$
$$= \lim_{u \to \infty} \frac{2 - \frac{1}{u} - \frac{1}{u^2}}{1 - \frac{1}{u} - \frac{2}{u^2}}$$
$$= \frac{2 - 0 - 0}{1 - 0 - 0}$$
$$= 2$$

Therefore, the horizontal asymptote is y = 2. The vertical asymptotes are found by setting what's in the denominator equal to zero and solving for x.

$$x^{2} + x - 2 = 0$$

 $(x + 2)(x - 1) = 0$
 $x = -2$ or $x = 1$



The function is graphed versus x below with the asymptotes labelled.